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HEAT TRANSFER IN TURBULENT FLOW OF FLUIDS THROUGH SMOOTH AND ROUGH TUBES

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Abstract—An analysis of the mechanism of heat transfer in smooth and rough tubes is presented. The experimental verification was carried out by heating air and water in tubes of 33/26 mm diameter and of roughness ratio, r/e, equal to 26·39, 13·5 and 9·15, as well as in a smooth tube of the same diameter; the roughness was formed by a 60° triangular thread. The Reynolds number was varied from $4{\cdot}5 \times 10^3$ to $1{\cdot}45 \times 10^5$ and the Prandtl number from 0·71 to 5·52. The relation following from the presented theory was used with success also for correlating the results of other authors, who experimented in systems with various forms of roughness.

NOMENCLATURE†

- A, surface of interface $[L^2]$;
- C, C', C'', constants;
- C_p, isobaric heat capacity $[HT^2/ML\Theta]$;
- d, diameter of tube defined by equation (10) $[L]$;
- e, dimension of roughness in radial direction $[L]$;
- f, friction factor defined by equation (14);
- F, cross-sectional area of tube $[L^2]$;
- G, rate of flow of fluid $[ML/T^3]$;
- g, acceleration of gravity $[L/T^2]$;
- h, coefficient of heat transfer $[H/L^2T\Theta]$;
- k, thermal conductivity $[H/LT\Theta]$;
- L, length of tube $[L]$;
- Nu, Nusselt number at mean temperature;
- P, pressure $[M/LT^2]$;
- ΔP, pressure drop $[M/LT^2]$;
- Pr, Prandtl number at mean temperature;
- R, gas constant $[L/\Theta]$;
- r, radius of tube, $r = d/2$ $[L]$;
- Re, Reynolds number at mean temperature;
- Re_T, Reynolds number at mean temperature defined by means of friction velocity;
- t, temperature $[\Theta]$;
- $\bar{t}$, mean temperature $[\Theta]$;
- T, absolute temperature $[\Theta]$;
- V, volume of tube $[L^3]$;
- $v_{\lambda r}$ velocity fluctuations in radial direction $[L/T]$;
- v_λ, velocity in fluctuation of dimension λ $[L/T]$;
- u_m, mean velocity $[L/T]$;
- u^*, friction velocity, $u^* = u_m\sqrt{(f/8)}$ $[L/T]$;
- y, distance from wall $[L]$.

Greek symbols

- ε, energy dissipated per unit mass and unit time $[L^2/T^3]$;
- ϵ, eddy viscosity $[L^2/T]$;
- λ_0, local degree of turbulence $[L]$;
- μ, dynamic viscosity $[M/TL]$;
- ν, kinematic viscosity $[L^2/T]$;
- η, efficiency;
- ρ, density $[M/L^3]$;
- τ, time $[T]$;
- a, thermal diffusivity $[L^2/T]$.

Subscripts

- f, refers to film temperature;

† M, mass;
L, length;
T, time;
Θ, temperature;
H, heat unit.

p, refers to beginning of measuring tube;
k, refers to end of measuring tube;
s, refers to wall of tube.

AS IS WELL known, among the important parameters affecting the rate of heat and mass transfer between phases are the hydrodynamic conditions. Usually this effect is taken care of by including the Reynolds number in the appropriate equations. But in more complex systems, which are met quite often in chemical-engineering practice, it is not always obvious what values are to be taken for the quantities appearing in the Reynolds number, in order that the value of the latter be uniquely related to the hydrodynamic conditions in those parts of the system that are of primary interest from the point of view of heat and mass transfer, i.e. on the interfacial surface. It seems, therefore, suitable to formulate the problem by means of the following two questions:

(1) How does the rate of the process depend on the length and time scales of turbulence without any regard to external conditions;
(2) How do the length scale and time scale of turbulence depend on the external conditions.

In an earlier paper [1] there was given an analysis of this problem on the basis of which there was derived a relation in which the hydrodynamic conditions are expressed by means of the quantity λ_0, defined by the equation

$$\lambda_0 = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} \tag{1a}$$

or by the equation

$$\frac{\lambda_0 \cdot v_{\lambda_0}}{\nu} = 1 \tag{1b}$$

According to Kolmogorov [17] λ_0 is the local degree of turbulence. This implies that as a first approximation it is assumed that the length and time scale of turbulence can be characterized sufficiently by means of the quantities ε, ρ and μ. This assumption is made by Kolmogorov in the region of so called "universal equilibrium", in a turbulent medium at high values of the Reynolds number, for the case of homogeneous and isotropic turbulence.

The theory is based on the concept that the motion of a fluid at large Reynolds numbers is characterized both by the velocity fluctuations v_λ and by a length λ in which these velocity changes occur. We can thus attribute to these fluctuations a certain Reynolds number

$$Re_\lambda = \frac{v_\lambda \cdot \lambda}{\nu} \tag{A}$$

Provided Re_λ is sufficiently large, the fluctuating motion is unstable so that further fluctuations arise from them and this process continues until the value of Re_λ becomes so small that no further fluctuations arise. The kinetic energy of the fluid is successively transferred by fluctuations of lower orders to higher fluctuations without, however, any significant dissipation of energy occurring by the viscous forces. Only when a certain size of the fluctuations is reached, for which

$$Re_\lambda = 1 \tag{B}$$

we will have intense energy dissipation. Denoting the velocity fluctuations and the appropriate length for which equation (B) is valid as v_{λ_0} and λ_0 respectively, we obtain equation (1b). The relation between the velocity v_λ and the length λ for fluctuations of the order k is given by

$$v_{\lambda k} \sim (\varepsilon \lambda_k)^{\frac{1}{3}} \tag{C}$$

which is known as the Kolmogorov–Obuchov law and gives the relation between the local velocity, size of the fluctuations and energy dissipated per unit mass and time.

Combining relations (B) and (C) we obtain equation (1a). As far as the mechanism of heat transfer is concerned we shall start from the following assumptions:

(a) The fluid at the interface consists of vortex elements of dimensions λ_0 and the fluid velocity in them v_{λ_0} where the relations (B) and (C) are valid for these quantities. These vortices represent the actual resistance for heat transfer on the phase boundary.

(b) Within these vortex elements heat is transported by conduction.

Solving the appropriate partial differential equation for the boundary and initial conditions

$$t = \text{const. for } y = 0 \text{ and } \tau > 0$$
$$t = t^n \quad \text{for } y > 0 \text{ and } \tau = 0$$

which case corresponds to the conduction of heat in a layer of infinite depth, we obtain for the heat-transfer coefficient the following relation

$$h = 2\sqrt{(a/\pi\tau_e)} \qquad \text{(D)}$$

According to assumption (a) the time τ_e over which the part of the surface of the vortex element is in contact with the medium with which heat transfer occurs is thus given by the relation

$$\tau_e = \frac{\lambda_0}{\lambda_0} \qquad \text{(E)}$$

Combining relations (B), (D) and (E) and rearranging we obtain

$$\frac{h\lambda_0}{k} = \frac{2}{\sqrt{(\pi)}}\left(\frac{\nu}{a}\right)^{0\cdot5} \qquad \text{(F)}$$

Since in addition to h and λ_0 equation (F) contains only the physical properties of the fluid, it is not limited to a particular type of system and should be valid generally.

Assuming homogeneous and isotropic turbulence in a tube of diameter, d, we can derive from equation (C) the following expression for λ_0

$$\lambda_0 \sim \frac{d}{Re^{\frac{3}{4}}} \qquad \text{(G)}$$

and after substituting this result in equation (F) we obtain

$$\frac{hd}{k} \sim Re^{\frac{3}{4}}\left(\frac{\nu}{a}\right)^{0\cdot5} \qquad \text{(H)}$$

which corresponds to the Reynolds analogy for $(\nu/a) = 1$.

Although it is apparent that the assumption of isotropy becomes less justified as we approach the phase boundary it may be expected that the transport mechanism of a scalar quantity on the phase boundary will be affected by the same quantities.

It will however be necessary to resort to experimental results concerning the appropriate turbulent field or make certain assumptions.

In the following we shall assume the validity of equation (F) which we modify as follows

$$\frac{hd}{k} = \frac{2}{\sqrt{\pi}}\left(\frac{d}{\lambda_0}\right)\left(\frac{C_p\,\mu g}{k}\right)^{0\cdot5} \qquad (2)$$

Thus we may consider the first part of the problem as solved and deal now with the second question, which is purely hydrodynamical, i.e. the determination of λ_0 for various conditions.

In the solution of this part of the problem we have to adopt a suitable procedure for each particular case. But we can take advantage of the fact, especially in more complex systems, that it is possible to reduce the problem to the determination of ϵ, the amount of energy dissipated per unit mass and unit of time in the region close to the phase boundary. On adopting certain simplifications for the model, the determination of ε is often relatively simple and does not require special measurements, because the pressure drop, which is the quantity we need to know, has to be measured for each system in any case, in order to define it. Although in this procedure we also have to adopt certain simplifications, as is the case to a larger or lesser degree in other methods of solution, it should be noted that for the cases to which the present theory has been applied [1, 2] the derived relations describe the processes with a precision that is often higher than that of special equations obtained in a more or less empirical way for a particular set of data.

A suitable case for demonstrating the above considerations is the transfer of heat in a tube. For smooth tubes, as well as for tubes whose relative roughness is very small, the effect of the hydrodynamic conditions can be expressed by means of the Reynolds number containing the mean velocity of flow, the tube diameter and the values of the physical properties of the fluid corresponding to the mean temperature. For higher values of the relative roughness this definition of the Reynolds number is no longer suitable and the course of the relations $Nu = f(Re, Pr)$ varies with the relative roughness.

Since tubes of circular cross section are amongst the most widespread elements in chemical engineering apparatus and the hydrodynamics and heat transfer in such tubes have been dealt with in a large number of articles, theoretical as well as experimental, it seemed advantageous to verify the new theoretical considerations on this case. Relatively little has been published so far on the effect of roughness on the rate of heat transfer in tubes. and the data of different authors disagree either partly or completely.

For these reasons, as well as because of the practical significance of the effect of wall roughness on heat transfer, an experimental investigation of the problem was carried out.

SURVEY OF LITERATURE

One of the earliest papers dealing with the effect of roughness on the rate of heat transfer is that of Soennecken [3], who found that the value of the coefficient of heat transfer is lower in rough tubes than that in smooth tubes. Stanton [4] obtained a contrary result. The discrepancy in the experimental results, as well as inconsistencies in the theory, led Pohl [5] to undertake systematic measurements of heat-transfer coefficients in smooth tubes, in tubes with normally rough walls, etched walls, and highly corroded walls. Pohl found that in all the rough tubes the heat-transfer coefficient was lower than in the smooth tube. No direct measure of the roughness is given in this work; the degree of roughness is only indicated by the pressure drop.

In order to avoid this uncertainty in the measure of roughness Cope [6] employed tubes on whose surface square pyramids were formed by a special procedure. From his experimental results Cope deduced that in the turbulent region wall roughness has a small effect on the value of the heat-transfer coefficient while in the transition region the effect is significant. The cited author was the first who employed in the correlation the friction velocity in place of the mean velocity, but no theoretical reasoning for this step is given.

On comparing smooth and rough tubes from an energetical point of view Cope arrived at the conclusion that for a given pressure drop smooth tubes are more efficient than rough tubes; the efficiency is defined as the ratio of energy transferred through the tube wall as heat to the energy consumed in the flow through the tube.

Sams [7] selected a different form of roughness; he experimented with tubes into which a square thread was cut. For correlating the data the friction velocity was used, similarly as in the work of Cope, but the values of the physical properties were taken at the film temperature.

Nunner [8] formed the wall roughness by placing into the tube at various distances spring rings of different forms and dimensions; in this way he obtained various types of roughness. The disadvantage of this procedure, as noted by the cited author, is that the rings do not form a single body with the wall; the error in the determination of h due to this fact may amount to 13 per cent. It was found that in the turbulent region the value of h in rough tubes may be as much as three times higher than that in smooth tubes, and the friction factor may increase up to fifteen times. For a description of the process Nunner used a generalized form of the Prandtl equation. Brouillette [9] determined experimentally the coefficient of heat transfer to water flowing in tubes with triangular grooves cut in the inside surface. He found that the values of h in the grooved tubes are from 10 to 100 per cent higher than in smooth tubes, and that the value of f is 50 per cent higher in the former. For a given value of h the pressure drop is smallest in smooth tubes.

Dipprey [10, 11] investigated the relation between friction and the rate of heat transfer in smooth and rough tubes. The rough tubes were obtained in the following manner: on a mandrel, whose surface was coated with a layer of granular material, nickel was deposited electrolytically and then the mandrel was dissolved. Dipprey found that the values of h increased up to 270 per cent and that f also increases considerably. There was also studied the effect of various expanding elements [12, 13] but these do not fall into the type of system considered in the present work.

THEORETICAL

As has been noted above, in order to be able to calculate the heat-transfer coefficient from the previously [1] derived equation (2) we need to know, for the system under consideration, the relation for calculating λ_0. In the present case we shall adopt a procedure differing from that employed previously [1] in that we shall make use of experimental results obtained in the measurement of turbulence in tubes [14]. λ_0 is interpreted as the size of eddies for which equation (1b) is valid; this condition can also be written in the form

$$\frac{\epsilon}{\nu} = 1. \tag{3}$$

The left-hand side of equation (3) can be approximated by means of a power function [15]

$$\frac{\epsilon}{\nu} = \left(\frac{u^* y}{\nu}\right)^n \tag{4}$$

Combining equations (3) and (4) we obtain

$$\frac{u^* y}{\nu} = 1. \tag{5}$$

A comparison of the left-hand sides of equations (1b) and (5) gives after rearrangement

$$\lambda_0 = \frac{u^*}{v_{\lambda_0}} y. \tag{6}$$

From the experimental results of Laufer [14] there follows for the region adjacent to the walls

$$\frac{v_\lambda}{u^*} = C \frac{u^* y}{\nu}. \tag{7}$$

On substituting into equation (1b) we obtain

$$\lambda_0 = \frac{1}{C} \cdot \frac{\nu}{u^*}. \tag{8}$$

Substituting from equation (8) for λ_0 in equation (2) and multiplying by d, we obtain after rearrangement

$$\frac{hd}{k} = C \frac{2}{\sqrt{\pi}} \left(\frac{u^* d}{\nu}\right) \frac{C_p \mu g}{k}\Big)^{0 \cdot 5} \tag{9}$$

Because the turbulence is not isotropic the value of the constant C will be different for the respective components of the fluctuations. For the case under consideration the most important component of the fluctuations will certainly be that in the radial direction; extrapolating the experimental results of Laufer we obtain for this component the value of C in equation (7)

$$C = \frac{1}{28}.$$

After substituting this value into equation (8) and (2) we obtain

$$\frac{hd}{k} = 0{\cdot}04 \left(\frac{u^* d}{\nu}\right) \left(\frac{C_p \mu g}{k}\right)^{0 \cdot 5} \tag{9a}$$

Equations (9) and (9a) were derived for a smooth tube, but as the relation for λ_0 contains only ν and u^* it may be expected that they will be valid also for tubes with rough walls. This point was verified experimentally.

EXPERIMENTAL

Description of apparatus

The heat-transfer coefficient was determined in horizontal brass tubes, 33 mm outer diameter, 26 mm inner diameter, and 800 mm long. The roughness was formed by cutting 60° triangular threads on the tube inside. The form of the thread is shown schematically in Fig. 1 and the relevant dimensions for all the investigated cases are given in Table 1. Some measurements were also carried out in smooth tubes. The arrangement of the apparatus is shown in Fig. 2.

The measuring tube *a* was placed in an insulated jacket of 100 mm diameter. Steam from the boiler *b* was introduced at the midpoint of the jacket; surplus steam was returned to the boiler via the condenser *c*, which was open to the atmosphere. The steam that condensed on

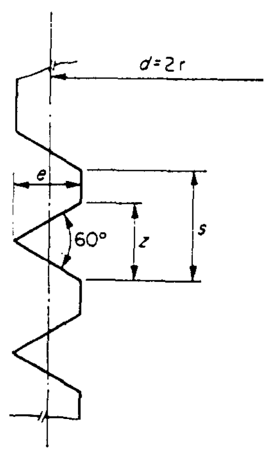

FIG. 1. View of rough surface of tubes.

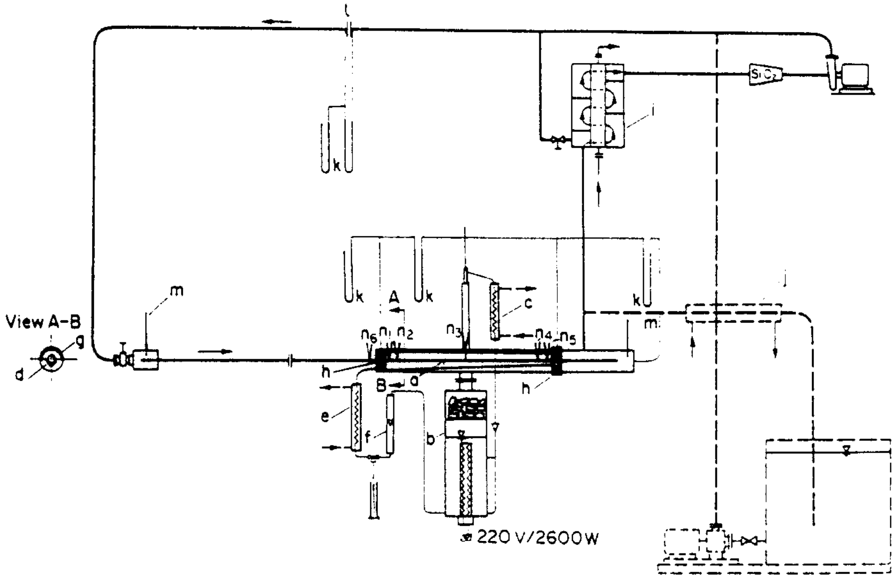

FIG. 2. Schematic arrangement of experimental unit:

a—measuring tube	*h*—housing for pressure tap
b—boiler	*i*—cooler for air
c—condenser	*j*—cooler for water
d—trough	*k*—U-manometer
e—cooler for condensate	*l*—orifice flowmeter
f—rotameter	*m*—mercury thermometers
g—cover	*n*—thermocouples.

Table 1. Dimensions of tubes

Tube No.	Depth of thread *e*	Pitch of thread *s*	Width of thread *z*	*d*	*r/e*
0	—	—	—	26·00	∞
1	0·5	0·8	0·6	26·39	26·39
2	1·0	1·6	1·2	26·99	13·50
3	1·5	2·4	1·8	27·47	9·15

the measuring tube collected in the trough *d*, placed beneath it; the condensate from the trough was cooled in the cooler e and from there either led back to the boiler via the rotameter *f* or collected in a vessel. The amount of steam condensed on the measuring tube during a certain period of time was determined by weighing the condensate collected in the vessel. Above the measuring tube a semicircular cover *g* was placed, which prevented the condensate formed on the jacket from getting into the trough, and also aided in keeping the whole space around the measuring tube full of steam. Ahead of the measuring tube, another tube of 1020 mm length was placed, and similarly at the downstream end a 240 mm length was placed. This ensured that the velocity profile was fully developed throughout the length of the measuring tube and end effects were thus eliminated. Each end of the measuring tube together with the end of the appropriate extension tube was held in a textgumoid housing *h* with a 1 mm gap between the tube ends in both housings. The housings also carried pressure taps. The working fluid, air or water, was run in a closed circuit. In order to obtain steady state conditions, the air circuit contained the cooler *i* and the water circuit, the cooler *j*. To keep the temperature fluctuations within the desired limits, the coolers *i* and *j* were supplied with cooling water by a pump, and not directly from the water mains. The air circuit was also equipped with a bed of silica gel, which ensured that the air was kept dry throughout the work. Pressures were measured by means of U-manometers *k* and by means of a micromanometer. The flow of fluid was measured by means of orifice meters *l*. Temperatures were measured by means of mercury thermometers *m* (at the orifice meters and at the outlet from the measuring tube) with 0·1 degC per division of scale, and by means of six thermocouples *n* (at the entrance to the measuring tube and the wall temperatures). Copper–constantan thermocouples were used. The thermocouples placed in the tube wall were coated with insulating varnish and embedded in low-melting solder. The hot junctions were placed as close as possible to the inner surface of the tube, at the root of the roughness elements. The cold junctions were held at 0°C by being placed into a vessel containing an ice–water mixture that was agitated gently. The electric potential of the thermocouples was measured by means of a potentiometer. A turbulizing element was placed ahead of the thermometer at the exit from the measuring tube, in order that the bulk stream temperature should be measured.

A check on the calculation of the amount of heat transferred was possible by weighing periodically the collected condensate.

The boiler was heated either electrically (in work with air) or by steam (in work with water).

Experimental procedure

On setting the desired rate of fluid flow the heating of the boiler was started; the rate of flow of cooling water was adjusted until steady state conditions were attained. The state was considered as steady if for 20 min all the measured quantities were practically constant. After this the rate of flow was determined by reading the pressure drop across the orifice meter and the temperatures indicated by the mercury thermometer at the orifice meter and at the outlet from the measuring tube. Then the electrical potentials of the thermocouples were measured. During these measurements at steady state, the amount of condensate was also determined. On completing the set of measurements the rate of fluid flow was altered and the whole procedure was repeated. The Reynolds number was varied in these experiments from 4500 to 145 000 and the Prandtl number from 0·71 to 5·52. A summary of the experimental results is given in Table 2.

EVALUATION OF DATA

As follows from equation (9), by means of which it was desired to correlate the experimental data, in addition to the physical properties of the fluid we also have to know the values of *d*, *h* and *f*.

The definition of the diameter of the rough tube seems to be highly significant, as this quantity is used in the calculation of both *h* and *f*. But in correlating the data by means of equation (9), *d* affects only the determination of *f*, because the Nusselt number contains *d* and *h*, a function of *d* (which is used in the calculation of the interfacial area); the form of the function $h = f(d)$ is such that the effect of *d* on the value of the Nusselt number cancels out.

Although *f* depends on the fifth power of *d*, the effect of the choice of *d* on the other parameter in equation (9), Re_T is also not very large as it depends on $d^{1\cdot5}$ and the maximum variation in *d* is 2*e*.

For these reasons the main requirement on the definition of *d* is that it be an easily measured quantity. In the present work the definition of *d* used by most authors [6–10] was adopted:

$$d = \sqrt{\left(\frac{4V}{\pi L}\right)} \qquad (10)$$

On the basis of the theoretical approach made in the present work it can be shown that the adopted definition of *d* is justified also theoretically. If we use the definition of λ_0 given by equation (1b), then λ_0 depends on ε, the amount of energy dissipated per unit mass of the fluid. Consider for simplicity that the hydrodynamic regime in the tube can be classified as homogeneous turbulence; then for ε we have

$$\varepsilon = \frac{\Delta P\, F u_m}{V \rho} \qquad (11)$$

and, therefore, the linear dimension of the system whose volume is *V* has to be expressed by means of definition (10).

Table 2.

(a) experimental results for air

G (kg/h)	P_p (mm H_2O)	ΔP (mm H_2O)	t_p (°C)	t_k (°C)	$\bar{t}_s$ (°C)
		Tube 0			
135·19	10189·2	165·0	40·15	55·74	99·22
124·26	10206·2	141·0	39·80	55·68	99·26
115·58	10161·3	123·0	40·30	56·25	99·26
104·40	10130·8	101·5	40·65	56·72	99·27
92·57	10101·8	81·0	40·25	56·79	99·27
75·90	10118·9	55·0	38·10	55·99	99·49
53·89	10075·4	29·0	37·55	56·83	99·53
92·42	10142·1	83·20	38·35	55·27	99·36
85·55	10123·1	71·90	38·95	55·92	99·38
78·01	10097·2	60·60	39·30	56·54	99·40
70·75	10079·8	50·30	39·25	56·88	99·41
62·83	10150·5	40·50	36·35	55·54	99·65
52·49	10129·0	29·40	36·90	56·49	99·67
39·68	10087·7	17·50	46·85	57·49	99·65
26·01	10066·2	7·30	36·10	58·10	99·68
28·49	10094·5	10·90	35·35	56·79	99·62
27·21	10088·5	9·95	35·40	57·15	99·62
25·64	10082·5	8·85	35·55	57·45	99·63
24·04	10077·9	7·80	35·45	57·70	99·64
22·46	10129·7	6·75	33·80	56·72	99·80
20·93	10117·6	5·80	34·30	57·17	99·79
18·61	10111·9	4·55	34·65	57·70	99·79
15·58	10101·4	3·05	35·05	58·52	99·80
12·46	10034·5	1·90	33·40	57·52	99·66
8·20	10029·5	0·80	35·65	57·52	99·69
71·44	10090·5	52·10	40·00	57·18	99·37
58·87	10063·5	36·70	39·70	57·86	99·38
43·82	10005·8	21·20	39·35	58·63	99·39
29·04	9986·1	10·10	38·25	59·11	99·41
		Tube 1			
124·82	10372·7	246·5	34·00	59·65	98·91
117·90	10331·7	212·5	34·43	60·02	99·10
103·93	10288·7	173·5	34·30	60·10	99·20
85·37	10233·6	118·0	34·32	60·45	99·31
71·00	10212·9	82·5	33·85	60·71	99·42
56·76	10172·9	53·5	33·70	61·16	99·47
36·99	10126·4	22·0	33·27	61·47	99·54
89·39	10286·6	133·0	36·07	61·46	99·14
81·07	10249·4	111·0	33·70	60·08	99·23
72·95	10222·9	90·5	34·05	60·51	99·29
63·71	10192·4	69·0	34·30	61·00	99·37
51·94	10129·9	48·0	33·80	61·21	99·39
38·57	10098·2	26·3	33·55	61·58	99·43
26·39	10073·4	11·3	33·25	61·21	99·50
29·05	10091·0	17·00	31·65	60·34	99·36
28·25	10082·4	16·00	31·85	60·50	99·38
27·44	10081·3	15·00	31·95	60·58	99·38
26·64	10078·2	14·05	32·05	60·62	99·37
25·52	10073·8	12·70	32·05	60·60	99·41
24·36	10084·4	11·65	31·85	60·45	99·47
23·30	10081·3	10·45	32·45	60·82	99·51
22·09	10076·4	9·25	35·40	62·33	99·42
20·88	10055·9	8·10	35·45	62·19	99·38
19·41	10049·5	6·75	35·85	62·19	99·41
18·13	10048·7	5·80	35·95	62·04	99·42
16·44	10022·6	4·55	36·00	61·70	99·39
14·81	10019·4	3·55	36·25	61·43	99·37
13·20	10012·4	2·75	36·40	61·06	99·41
10·66	10004·4	1·60	37·85	60·88	99·44
7·20	10000·6	0·60	40·70	59·83	99·44
		Tube 2			
110·37	10247·1	290·5	41·00	68·54	98·30
100·48	10218·1	238·0	40·05	68·24	98·45
87·27	10260·6	174·5	38·10	67·54	98·65
78·49	10226·1	141·5	38·85	68·22	98·79
63·22	10171·2	92·0	38·20	68·47	98·83
46·24	10118·2	49·0	37·00	68·99	99·07
83·38	10227·3	168·0	39·65	68·41	96·78
78·35	10207·3	148·0	39·70	68·64	98·45
73·68	10185·0	130·5	39·95	68·94	98·59
68·73	10168·0	113·5	40·05	69·16	98·68
62·84	10130·7	95·5	38·15	68·34	98·59
55·18	10104·7	74·5	38·50	68·92	98·64
47·24	10074·5	55·5	38·45	69·45	98·91
37·60	9977·2	34·5	35·75	68·63	98·86
24·33	9952·9	14·5	34·50	69·42	99·02
28·40	10141·6	22·80	34·95	69·10	99·20
26·85	10136·8	20·40	35·55	69·65	99·47
25·10	10112·1	17·85	36·30	70·26	99·16
23·06	10081·1	15·30	36·05	70·30	99·41
21·87	10062·8	13·90	37·00	70·94	99·33
20·51	10060·1	12·25	37·35	71·25	99·23
19·37	10099·6	10·75	34·35	69·81	99·12
18·05	10094·5	9·40	35·20	70·59	99·17
16·58	10091·0	7·95	35·45	70·71	99·17
14·85	10092·3	6·25	33·00	69·46	99·38
13·01	10088·8	4·65	33·55	69·65	99·43
10·08	10105·3	2·55	32·75	67·77	99·44
6·74	10101·9	0·85	34·70	63·35	99·48
		Tube 3			
112·04	10400·3	319·0	39·10	67·84	99·03
100·01	10385·3	251·0	38·55	67·85	99·21
88·21	10281·3	195·0	37·95	67·67	99·08
79·50	10238·8	157·0	37·95	68·11	99·18
66·45	10144·7	109·0	37·80	68·52	99·18
48·02	10079·7	57·0	37·60	69·29	99·28
83·67	10328·8	182·0	37·70	67·79	99·23
79·44	10309·3	164·5	37·55	67·86	99·29
72·55	10274·3	136·0	37·80	68·33	99·34
66·71	10233·4	116·0	38·05	68·59	99·31

Table 2—continued

(*a*) *experimental results for air—continued*

G (kg/h)	P_p (mm H_2O)	ΔP (mm H_2O)	t_p (°C)	t_k (°C)	$\bar{t}_s$ (°C)
		Tube 3—*continued*			
58·93	10202·4	90·5	38·10	69·00	99·34
53·13	10180·9	74·0	37·85	69·08	99·40
46·50	10171·6	55·7	36·15	68·62	99·50
38·93	10150·6	39·1	36·55	69·26	99·56
25·93	10118·7	16·6	36·85	70·20	99·66
28·69	10207·5	24·25	35·70	69·33	99·76
26·96	10201·4	21·25	36·05	69·62	99·79
25·21	10193·2	18·80	36·55	70·05	99·77
23·23	10189·3	16·05	37·05	70·39	99·81
22·04	10212·5	14·35	36·15	70·03	99·85
20·88	10202·8	12·75	36·65	70·42	99·86
19·57	10201·5	11·30	37·35	70·88	99·87
18·30	10182·0	9·80	37·80	70·15	99·82
16·76	10179·1	8·25	37·90	72·23	99·82
15·25	10196·2	6·70	37·35	70·85	99·89
13·20	10194·1	5·00	38·15	71·20	99·90
10·91	10185·6	3·35	39·15	71·18	99·90
7·54	10181·7	1·35	41·95	69·20	99·92

(*b*) *experimental results for water*

G (kg/h)	ΔP (mm H_2O)	t_p (°C)	t_k (°C)	$\bar{t}_s$ (°C)
		Tube 1		
6198·2	639·4	33·48	36·47	66·01
5773·6	556·2	33·77	36·83	66·46
5267·6	470·0	36·35	40·20	70·80
4717·2	381·0	36·45	40·92	71·57
4297·9	316·3	34·82	39·43	70·81
3288·8	176·4	33·01	38·85	71·47
2280·2	86·6	29·99	37·59	72·54
2091·1	69·3	29·39	38·19	74·65
1870·4	58·8	28·67	38·55	76·26
1622·4	44·6	27·36	38·35	78·22
1298·1	29·7	24·52	36·39	78·57
956·1	15·3	22·95	38·01	83·78
996·6	19·50	29·18	41·16	84·02
917·8	16·76	27·75	39·66	84·11
829·6	14·24	26·71	39·54	85·67
729·2	9·64	25·41	38·75	88·00
635·3	6·76	24·92	38·30	86·81
473·6	4·58	25·45	39·61	91·98
		Tube 2		
6070·6	847·0	34·52	37·91	68·72
5431·0	689·6	34·35	38·18	70·56
4832·1	530·6	31·65	35·85	69·66
3964·8	363·1	31·25	36·41	70·55
3240·9	230·5	30·25	36·46	72·05
2922·8	202·3	29·97	36·83	73·12
2529·5	136·4	29·22	37·23	74·12
2221·3	112·4	27·25	35·26	71·43
1888·6	76·4	25·75	34·97	74·02
1636·7	61·1	25·37	35·68	75·73
1391·8	38·8	24·95	37·28	77·10
985·3	24·20	25·60	41·44	83·32
926·4	21·85	25·30	41·61	83·74
868·2	16·90	25·40	42·75	84·56
807·0	15·60	25·20	43·72	86·63
736·4	14·05	24·35	44·20	86·02
647·6	12·30	23·75	44·81	87·08
532·5	8·15	22·65	45·82	88·19
424·3	5·30	20·90	47·28	91·00
		Tube 3		
6078·7	1040·6	49·77	52·47	74·41
6050·4	1021·9	40·77	44·55	72·42
5609·4	915·3	39·97	44·05	72·38
5099·3	727·3	39·20	43·62	73·00
4449·7	564·2	38·27	43·29	73·90
3731·6	385·4	36·95	42·78	73·86
3697·1	377·2	48·55	52·63	76·69
2992·3	237·4	35·70	42·91	74·91
2873·8	215·0	35·61	43·18	75·87
2712·1	193·9	35·52	43·56	76·70
2578·7	172·7	35·03	43·33	76·13
2433·0	152·7	35·05	44·13	78·35
2218·1	125·7	34·52	44·24	79·05
2001·9	102·2	34·55	45·43	80·12
1752·9	84·7	44·80	53·12	79·60
1764·9	75·2	33·67	45·83	81·73
1469·7	52·9	32·30	46·01	82·08
1085·4	24·7	30·55	48·37	86·09
998·8	29·28	40·81	55·08	85·37
1003·3	26·12	30·17	47·42	83·72
946·6	25·42	29·73	47·73	84·28
884·5	22·48	29·10	48·04	85·82
788·2	20·27	37·29	55·79	88·64
688·9	16·86	36·13	56·11	89·39
528·9	9·87	37·38	59·78	91·72
367·1	7·67	34·12	60·68	93·12

The heat-transfer coefficient was calculated from the relation

$$h = \frac{G \cdot C_p (t_k - t_p)}{A \cdot \Delta t} \tag{12}$$

where

$$\Delta t = \frac{t_k - t_p}{\ln \dfrac{\bar{t}_s - t_p}{\bar{t}_s - t_k}} \tag{13}$$

The friction factor f was calculated from the relation

$$f = \frac{\Delta P_T}{\dfrac{L}{d} \rho \dfrac{u_m^2}{2}} \tag{14}$$

where

$$\Delta P_T = \Delta P - \frac{G^2 \cdot R}{F^2 g} \left(\frac{T_k}{P_k} - \frac{T_p}{P_p} \right) \tag{15}$$

The physical quantities appearing in equations (12) and (14), and the remaining quantities in equation (1) were determined and evaluated for the mean temperature as well as for the film temperature, defined as

$$\bar{t}_f = \frac{\bar{t}_s + \bar{t}}{2} \tag{16}$$

The reason for adopting these two methods of evaluation is that the evaluation for the mean temperature is usually employed (and this will enable us to compare the present results with those of other authors), while the evaluation for the film temperature is more in accord with the physical model of the process, on the basis of which equation (9) was derived.

The data were correlated in the usual manner by means of the relation

$$\frac{Nu}{Pr^{0 \cdot 4}} = f(Re), \tag{17}$$

as shown in Fig. 3; they were further correlated by means of equation (9), using the film temperature (see Fig. 5) as well as the mean temperature; the data for the last case are plotted in Fig. 6. For comparison, data of other authors are also given in Fig. 6. A comparison of the results obtained for the film temperature in the present work with those of other authors was

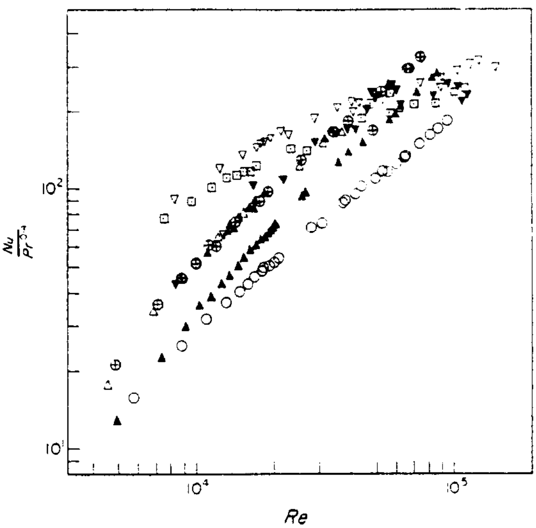

FIG. 3. Dependence of ($Nu/Pr^{0 \cdot 4}$) on Re according to equation (17).

Tube	0	1	2	3
Air	○	▲	△	⊕
Water		▼	⊡	▽

not possible because of lack of material. In Fig. 4 are shown plots of the dependence of f on Re for both the mean and film temperatures.

As is apparent from Fig. 4, the following empirical formulae can be obtained for the dependence of f on Re:

$$f = 0 \cdot 497 \left(\frac{e}{d} \right)^{0 \cdot 63}, \tag{18}$$

which is valid for both fluids at the mean temperature and for $Re > 2 \cdot 5 \times 10^4$, and

$$f_f = 0 \cdot 515 \left(\frac{e}{d} \right)^{0 \cdot 63} \tag{19}$$

valid for both fluids at the film temperature and for $Re_f > 3 \times 10^4$.

By means of a statistical treatment of the data shown in Fig. 5, i.e. for the film temperature, there was obtained the relation

$$\log \left(\frac{Nu}{Pr^{0 \cdot 5}} \right)_f = 0 \cdot 98622 \log Re_{Tf} - 1 \cdot 23553 \pm 0 \cdot 03771 \tag{20}$$

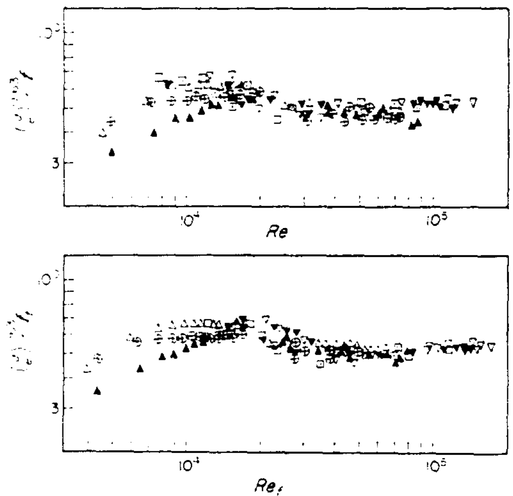

FIG. 4. Dependence of friction factor on Re:
(a) for mean temperature
(b) for film temperature

Tube	1	2	3
Air	▲	△	⊖
Water	▼	⊡	▽

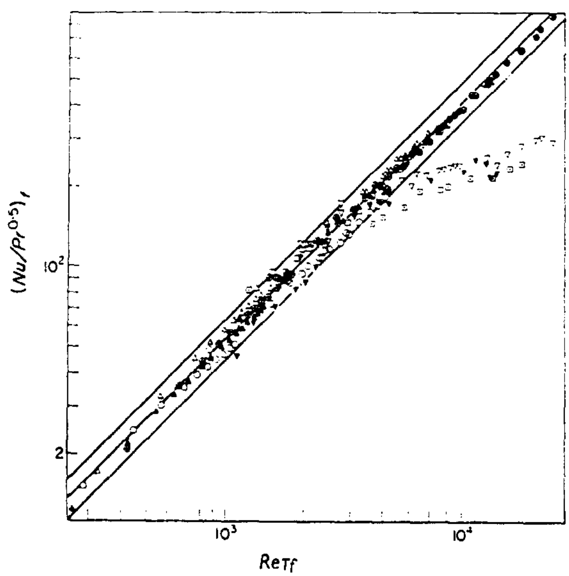

FIG. 5. Dependence of $(Nu/Pr^{0.5})_f$ on Re_{Tf} according to equation (9).

Tube	0	1	2	3
Air	○	▲	△	⊕
Water		▼	⊡	▽
Recalculated values		◐	◑	●

or, after transforming

$$Nu_f = 0{\cdot}05814\, Re_{Tf}^{0{\cdot}986}\, Pr^{0{\cdot}5}. \tag{20a}$$

For the set of 146 results the value of the correlation coefficient was 0·99. Equation (20a) can be approximated by the relation

$$Nu_f = 0{\cdot}0517\, Re_{Tf}\, Pr_f^{0{\cdot}5}. \tag{20b}$$

Within the range of values of Re from 2×10^3 to 10^5 the error introduced by the approximation does not exceed 4 per cent.

The agreement of the heat balance obtained from the rate of condensation on the tube and from the enthalpy increase of the fluid stream was on the average 3·5 per cent.

As is seen from Fig. 5, the experimental results for the coefficient of heat transfer into water agree with the assumed relation for Re_{Tf} smaller than about 3×10^3; the results for higher values of Re_{Tf} do not seem, on first sight, to correspond to the theory. A more detailed analysis of the system led to the conclusion that this may be due to the fact that the thermocouples inserted into the tube wall do not indicate the surface temperature of the roughness element, but approximately the temperature at its root. With gases, and at low values of Re_{Tf} with liquids, when the value of h is relatively small in comparison to the thermal conductivity of the wall, this will not show, because the temperature drop over the roughness element is small. But with liquids at higher values of Re_{Tf}, when the value of h is also high, the mentioned fact may be a source of considerable error in the calculation of h, because the surface temperature of the roughness element may vary considerably in the direction from its root to the top. In this case the roughness element acts as a fin.

The results for $Re_{Tf} > 3 \times 10^3$ were, therefore, recalculated under the simplifying assumptions that the roughness element may be considered as a straight fin of the same form, for which we know the temperature at the base and h', corresponding to h from equation (20a) at the particular value of Re_{Tf}. For a given temperature at the base and the value of h' following from equation (20a), the temperature at the top of the roughness element was calculated, and the mean value of these two temperatures was taken as the wall temperature $\bar{t}'_s$. The calculation of $\bar{t}'_s$ was checked by means of the equation

$$h/\ln\frac{\bar{t}_s - t_p}{\bar{t}_s - t_k} = h'/\ln\frac{\bar{t}'_s - t_p}{\bar{t}'_s - t_k}$$

The wall temperature obtained in this way is equal to the sum of the base and top temperatures divided by a factor whose value is on the average 2·1 instead of 2, as was assumed. The recalculated values for the corrected wall temperatures are also shown in Fig. 5. Because in these calculations values of h' obtained from equation (20a) were used, the spread of the recalculated values is less than that of the original data. In view of the approximations introduced in the calculation of $\bar{t}'_s$ or h', a more accurate procedure was not justified.

DISCUSSION OF RESULTS

In the first place it should be noted that the experimental results provide a practically complete proof of the validity of equation (9), since the exponents over Re_{Tf} and Pr_f agree very well with the theoretical values. Also the value of the constant in equation (20a) is close to that of the constant in equation (9). The effect of the Prandtl number was further investigated graphically; it was found that the best value of the exponent is 0·5, in agreement with the theory, and not the usually presented values of from 0·33 to 0·4.

Now we shall compare the results of the present work with those of other authors.

The usually recommended relation for smooth tubes, proposed by Dittus and Boelter [16],

$$Nu = 0{\cdot}023\ Re^{0{\cdot}8}\ Pr^{0{\cdot}4} \tag{21}$$

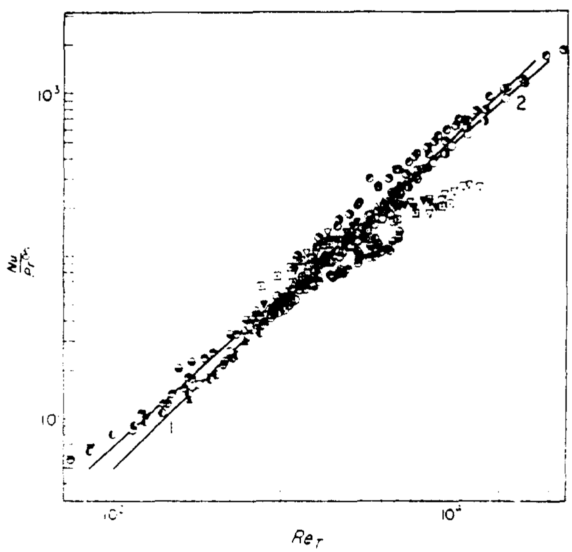

FIG. 6. Comparison of results of different authors.

Author	Fluid	Tube 0	1	2	3	A	B	C	E–3	A–4	C–9	D–3
Kolář	Air	○	▲	△	⊕							
	Water		▼	⊡	▽							
Cope	Water					◒	◓	◑				
Dipprey	Water								⊙	◔	◕	◑
Dittus–Boelter	Air		line 2—equation (23)									
Nunner	Air		line 1—equation (26)									

can be rearranged to the form of equation (9) in the following manner: first the right-hand side of equation (21) is multiplied by $Pr^{0.1}$ and the coefficient is divided by the mean value of $Pr^{0.1}$, thus we obtain on the right-hand side of the equation $Pr^{0.1}$, which is by division transferred to the left-hand side of the equation. Then Re_T is introduced by multiplying and dividing the resulting right-hand side by $[\sqrt{(f/8)}]^{0.8}$. For expressing f we make use of the Blasius relation

$$f = 0.316\, Re^{-1/4} \tag{22}$$

rearranged to the form $f = f(Re_T)$; from this we obtain $[\sqrt{(f/8)}]^{0.8}$ as a function of Re_T and substitute this into the adjusted equation (21). Thus we obtain

$$\frac{Nu}{Pr^{0.5}} = 0.102\, Re_T^{0.914} \tag{23}$$

A plot of this equation is given in Fig. 6, and as can be seen the agreement with the results of the present work is good, practically in the whole range of validity of equation (22). Sams [7] in his correlation also defines the Reynolds number by means of the friction velocity, similarly as Cope. The effect of Pr was not investigated by Sams, as he only worked with air, and for the exponent over Pr he took 0·4. Throughout the investigated region, the results of Sams are about 20 per cent lower than those of the present work as well as those of other authors. The results of Cope and Dipprey were recalculated in accordance with the parameters employed in this paper and are plotted together with the results of the present work in Fig. 6. As can be seen from the figure there is good agreement between the results obtained by different authors in experiments with different forms of roughness and at various conditions. The spread of the data could apparently be reduced if a correlation for the film temperature were possible; this is apparent from a comparison of the plots of the present results shown in Figs. 6 and 5. In some degree Fig. 6 can also serve as evidence for the reason given above why the data for liquids at higher values of Re_T deviate from the predicted course. The results of Cope begin to deviate from the predicted course at the lowest values of Re_T; the form of the roughness elements with which these results were obtained, rectangular pyramids, evidently displays a strong tendency to intensive cooling in the direction of the element axis. Next to deviate are the results of the present work, where a less pronounced tendency to cooling exists for the employed roughness elements. The last to deviate are the results of Dipprey, who experimented with roughness of a form for which it can be assumed that the tendency for cooling along the depth of the elements is even less pronounced.

As has been mentioned above, Soennecken [3] and Pohl [5], arrived at the conclusion that the value of the heat-transfer coefficient in rough tubes is less than in smooth tubes. This is most probably due to the fact that the thermal expansion of the tube could not serve as a reliable basis for evaluating the actual temperature of the internal surface of the tube, and as they used a liquid for the working fluid (water), they arrived at an erroneous conclusion. In principle, we have here the same effect as that which produces the deviation of the results for water at higher values of Re_T, as was discussed above.

The results of Nunner [8] can be compared with those of the present work on the basis of equation (50), given in the cited paper [8]:

$$Nu = 0.383\, Re^{0.68} f^{1/m} \tag{24}$$

where $m = (Re/100)^{\frac{1}{4}}$. For the range of values of the Reynolds number from 10^4 to 4×10^4. Nunner gives for the exponent over f, i.e. $1/m$, the mean value 0·5. If equation (24) is rearranged to the form used in the present work, taking $Pr = 0.72$, we obtain for the above range

$$\frac{Nu}{Pr^{0.5}} = \frac{1.75\, Re_T}{Re^{0.32}} \tag{25}$$

For the mean value of the interval, i.e. for $Re = 2.5 \times 10^4$, we obtain

$$\frac{Nu}{Pr^{0.5}} \approx 0.05\, Re_T \tag{26}$$

which is in very good agreement with the results of the present work, as can be seen from Fig. 6.

In order to be able to estimate the suitability of surface roughness as a means for increasing the heat-transfer coefficient we have to consider the energy balance and compare the amount of energy transferred as heat per unit temperature difference with the amount of energy needed for passing the fluid through the tube

$$\eta = \frac{hA}{\Delta P \cdot F \cdot u_m} \tag{27}$$

Substituting for A and F, further for h from equation (20b), and for ΔP from the Fanning equation (14), we obtain on rearrangement

$$\eta = \frac{0{\cdot}0517\, C_p}{Pr^{0{\cdot}5}} \cdot \frac{1}{\sqrt{(f/8)}\, u_m^2}. \tag{28}$$

For comparing the smooth and rough tube we consider a fluid having the same physical properties and flowing at the same rate in both cases; equation (28) thus reduces to

$$\eta = C \frac{1}{\sqrt{(f/8)} \cdot u_m^2} \tag{28a}$$

For smooth tubes we obtain by means of the Blasius equation (22)

$$\eta = C' \frac{1}{v_m{}^{15/8}} \tag{28b}$$

where

$$C' = C \frac{G^{\frac{1}{8}}}{0{\cdot}195\, \nu^{\frac{1}{8}}}$$

In rough tubes at high values of Re the friction factor f is constant, and, therefore, for a given relative roughness equation (28) reduces to

$$\eta = C'' \frac{1}{u_m^2}, \tag{28c}$$

where

$$C'' = \frac{C}{0{\cdot}718\, (e/d)^{0{\cdot}315}}$$

On comparing equations (28b) and (28c) we see that the efficiency of rough tubes decreases with increasing velocities only somewhat more rapidly than that for smooth tubes, and that at a given velocity the efficiency of a smooth tube is always higher than that of a rough tube, as follows from equation (28a). From equation (28) it is seen that for constant values of $\sqrt{(f/8)}$ and v, the efficiency is directly proportional to C_p and inversely proportional to $Pr^{0{\cdot}5}$.

It is apparent that the introduced efficiency cannot be taken as a sufficient criterion for the design of a heat exchanger, as it includes neither the first cost of the exchanger nor the operating cost for obtaining the desired difference in temperatures between the heated fluid and wall. But since both costs depend on local factors, it would not be suitable to pursue in the present paper the discussion of the problem beyond the general statement already given.

In conclusion it may be noted that a relation of the form (9) depicts well the mechanism of heat transfer for gases and very probably also for liquids in smooth tubes as well as in tubes with various forms of surface roughness, if the actual temperature of the interface is known and if the values of the physical properties are taken for the film temperature.

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Zusammenfassung—Für den Mechanismus des Wärmeübergangs in glatten und rauhen Rohren wird eine Analyse vorgelegt. Die experimentelle Verwirklichung wurde erzielt, indem Luft bzw. Wasser in rauhen Rohren von 33 bzw. 26 mm Durchmesser und einem Rauhigkeitsverhältnis r/e von 26,39, 13,5 und 9,15 und in glatten Rohren vom gleichen Durchmesser erhitzt wurde. Die Rauhigkeit wurde durch ein 60° Dreiecksgewinde vorgegeben, Die Reynoldszahl wurde von $4,5 \times 10^3$ bei $1,45 \times 10^5$ und die Prandtlzahl von 0,71 bis 5,57 variiert, Die Beziehung, die sich aus der vorgelegten Theorie ergab, wurde mit Erfolg auch dazu verwendet, die Ergebnisse anderer Autoren, die an Systemen mit verschniedenartigen Rauhigkeiten experimentierten, zu korrelieren.

Аннотация—Рассматривается теплообмен в трубах с гладкими и шероховатыми стенками. Экспериментальная проверка проводилась путем нагревания воздуха и воды в трубах диаметром 33/26 mm и с коэффициентами шероховатости $r/e = 26{,}39$, 13,5 и 9,15, а также в гладких трубах того же диаметра; шероховатость создавалась 60° треугольной резьбой. Число Рейнольдса изменялось в диапазоне от $4{,}5 \times 10^3$ до $1{,}45 \times 10^5$, а число Прандтля от 0,71 до 5,52. Зависимость, вытекающая из приведенного анализа, с успехом использовалась также для обобщения результатов работ других авторов, проводивших эксперименты с системами с различными видами шероховатости.